Ex) Apply Jacobi's method to solve:

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$

Sol^n:- The above system is diagonally dominant i.e. in each eq^n the absolute value of the largest co-efficient is greater than the sum of the remaining co-efficients. The given eq^{n}s can be written as,

$$x = \frac{12}{10} - \frac{y}{10} - \frac{z}{10}$$
$$y = \frac{13}{10} - \frac{2x}{10} - \frac{z}{10}$$
$$z = \frac{14}{10} - \frac{2x}{10} - \frac{2y}{10}$$

we start the iteration by putting $x=0, y=0, z=0$.

$\therefore$ For the first iteration, we get,

$$x^{(1)} = \frac{12}{10}$$
$$y^{(1)} = \frac{13}{10}$$
$$z^{(1)} = \frac{14}{10}$$

Again,

$$x^{(2)} = \frac{12}{10} - \frac{y^{(1)}}{10} - \frac{z^{(1)}}{10}$$

$$= \frac{12}{10} - \frac{13}{10\times10} - \frac{14}{10\times10}$$

$$= \frac{12}{10} - \frac{13}{100} - \frac{14}{100}$$

$$= \frac{93}{100}$$

$$y^{(2)} = \frac{13}{10} - \frac{x^{(1)}}{5} - \frac{z^{(1)}}{10}$$

$$= \frac{13}{10} - \frac{12}{10\times5} - \frac{14}{10\times10}$$

$$= \frac{92}{100}$$

$$z^{(2)} = \frac{14}{10} - \frac{x^{(1)}}{5} - \frac{y^{(1)}}{5}$$

$$= \frac{14}{10} - \frac{12}{10\times5} - \frac{13}{5\times10}$$

$$= \frac{45}{50} = \frac{90}{100}$$

Repeating the process with $x^{(2)}, y^{(2)}, z^{(2)}$, we get,

$$x^{(3)} = \frac{12}{10} - \frac{y^{(2)}}{10} - \frac{z^{(2)}}{10}$$

$$= \frac{12}{10} - \frac{92}{10\times100} - \frac{90}{10\times100}$$

$$= \frac{1200-92-90}{1000} = \frac{1018}{1000} = 1.018$$

$$y^{(3)} = \frac{13}{10} - \frac{x^{(2)}}{5} - \frac{z^{(2)}}{10}$$

$$= \frac{13}{10} - \frac{93}{5\times100} - \frac{90}{100\times10}$$

$$= \frac{1300-186-90}{1000} = 1.024$$

$$z^{(3)} = \frac{14}{10} - \frac{x^{(2)}}{5} - \frac{y^{(2)}}{5}$$

$$= \frac{14}{10} - \frac{93}{5\times100} - \frac{92}{5\times100}$$

$$= \frac{515}{500} = 1.03$$

Similarly,

$$x^{(4)} = \frac{12}{10} - \frac{y^{(3)}}{10} - \frac{z^{(3)}}{10}$$

$$= \frac{12}{10} - \frac{1.024}{10} - \frac{1.03}{10}$$

$$= \frac{9.946}{10} = .9946$$

$$y^{(4)} = \frac{13}{10} - \frac{x^{(3)}}{5} - \frac{z^{(3)}}{10}$$

$$= \frac{13}{10} - \frac{1.018}{5} - \frac{1.03}{10}$$

$$= \frac{\cancel{13}\ 9.934}{10} = .9934$$

$$z^{(4)} = \frac{14}{10} - \frac{x^{(3)}}{5} - \frac{y^{(3)}}{5}$$

$$= \frac{14}{10} - \frac{1.018}{5} - \frac{1.024}{5}$$

$$= \frac{9.916}{10} = .9916$$

Proceeding in this way, we obtain $x=1$, $y=1$ and $z=1$ as the correct appropriate values.

Ex) Solve the following system of eq^n s by Gauss-Seidel:

$$10x_1 + x_2 + x_3 = 12$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$2x_1 + 2x_2 + 10x_3 = 14$$

Sol^n: From the given eq^n s, we have,

$$x_1 = \frac{1}{10}(12 - x_2 - x_3) \quad --- (1)$$
$$x_2 = \frac{1}{10}(13 - 2x_1 - x_3) \quad --- (2)$$
$$x_3 = \frac{1}{10}(14 - 2x_1 - 2x_2) \quad --- (3)$$

Putting $x_2 = 0$ and $x_3 = 0$ in the right side of (1), we get,

$$x_1^{(1)} = \frac{1}{10} \times 12 = 1.2$$

Putting $x_1 = x_1^{(1)} = 1.2$, $x_3 = 0$ in (2), we get

$$x_2^{(1)} = \frac{1}{10}(13 - 2 \times 1.2 - 0)$$
$$= 1.06$$

Putting $x_1 = x_1^{(1)} = 1.2$ and $x_2 = x_2^{(1)} = 1.06$ in (3) we get,

$$x_3^{(1)} = \frac{1}{10}(14 - 2 \times 1.2 - 2 \times 1.06)$$
$$= 0.948$$

For the 2nd iteration, we have,

$$x_1^{(2)} = \frac{1}{10}(12 - x_2^{(1)} - x_3^{(1)})$$
$$= \frac{1}{10}(12 - 1.06 - 0.948)$$
$$= 0.9992$$

$$x_2^{(2)} = \frac{1}{10}\left(13 - 2x_1^{(2)} - x_3^{(1)}\right)$$

$$= \frac{1}{10}\left(13 - 2\times 0.9992 - 0.948\right)$$

$$= 1.00536$$

$$x_3^{(2)} = \frac{1}{10}\left(14 - 2x_1^{(2)} - 2x_2^{(2)}\right)$$

$$= \frac{1}{10}\left(14 - 2\times 0.9992 - 2\times 1.00536\right)$$

$$= 0.999088$$

For the 3rd iteration, we have,

$$x_1^{(3)} = \frac{1}{10}\left(12 - x_2^{(2)} - x_3^{(2)}\right)$$

$$= \frac{1}{10}\left(12 - 0.9992 - 1.00536\right)$$

$$= .999544$$

$$x_2^{(3)} = \frac{1}{10}\left(13 - 2x_1^{(3)} - x_3^{(2)}\right)$$

$$= \frac{1}{10}\left(13 - 2\times 0.999544 - 0.999088\right)$$

$$= 1.0001824$$

$$x_3^{(3)} = \frac{1}{10}\left(14 - 2x_1^{(3)} - 2x_2^{(3)}\right)$$

$$= \frac{1}{10}\left(14 - 2\times .999544 - 2\times 1.0001824\right)$$

$$= 1.00005472$$

The values of x_1, x_2, x_3 at the end of the 3rd iteration are

$$x_1 = .9995$$

$$x_2 = 1.0002$$

$$x_3 = 1.0000$$

Continuing this way further iterations can be obtained.