Theorem: Let $n > 1$ be fixed and a, b, c, d be arbitrary integers. Then the following properties hold:

(i) $a \equiv a \pmod{n}$

(ii) If $a \equiv b \pmod{n}$, then $b \equiv a \pmod{n}$

(iii) If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.

(iv) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $a + c \equiv b + d \pmod{n}$ and $ac \equiv bd \pmod{n}$

(v) If $a \equiv b \pmod{n}$, then $a + c \equiv b + c \pmod{n}$ and $ac \equiv bc \pmod{n}$

(vi) If $a \equiv b \pmod{n}$, then $a^k \equiv b^k \pmod{n}$ for any positive integer k.

Pf (i) For any integer a, we have

$$a - a = 0 = 0 \cdot n$$
$$\Rightarrow n \mid (a - a) \Rightarrow a \equiv a \pmod{n}$$

(ii) Let $a \equiv b \pmod{n}$

$$\Rightarrow m \mid (a - b)$$
$$\Rightarrow a - b = mk \text{ for some integer } k.$$
$$\therefore b - a = -(a - b) = -mk = m(-k) = mk', \text{ where } k' = -k \in \mathbb{Z}$$