*** Newton-Gregory backward difference interpolation formula:**

Let $y=f(x)$ be the given function. Assuming that $f(x)$ can be represented by a polynomial of nth degree in x, and expressing it in factorial notation, we suppose

$$f(x) = A_0 + A_1[x-(a+nh)]^{(1)} + A_2[x-(a+nh)]^{(2)} + \cdots$$
$$+ A_n[x-(a+nh)]^{(n)} \quad \cdots\cdots (1)$$

where $A_0, A_1, A_2, \ldots, A_n$ are constants.

$$\therefore\ f(x) = A_0 + A_1[x-(a+nh)] + A_2[x-(a+nh)][x-\{a+(n-1)h\}]$$
$$+ \cdots + A_n[x-(a+nh)] \cdots [x-(a+h)]$$

Putting, $x = a+nh$

$(1) \Rightarrow \quad f(a+nh) = A_0$

for $x = a+(n-1)h$

$(1) \Rightarrow$

$$f\{a+(n-1)h\} = A_0 + A_1(-h)$$

$$\Rightarrow A_1 = \frac{f(a+nh) - f(a+nh-h)}{h}$$

$$\Rightarrow A_1 = \frac{\nabla f(a+nh)}{h}$$

for $x = a+(m-2)h$.

(1) $\Rightarrow$

$$f\{a+(m-2)h\} = A_0 + A_1(-2h) + A_2(-2h)(-h)$$

$$\Rightarrow A_2 = \frac{f\{a+(m-2)h\} + f(a+mh) - 2f\{a+(m-1)h\}}{2h^2}$$

$$\Rightarrow A_2 = \frac{\nabla^2 f(a+mh)}{\lfloor 2\, h^2}$$

and so on.

Thus $A_m = \dfrac{\nabla^m f(a+mh)}{\lfloor m \cdot h^m}$

Substituting the values of $A_0, A_1, A_2, \ldots A_m$ in (1), we get

$$f(x) = f(a+mh) + \frac{\nabla f(a+mh)}{h}[x-(a+mh)] + \cdots$$

$$+ \frac{\nabla^m f(a+mh)}{\lfloor m \cdot h^m}[x-(a+mh)][x-\{a+(m-1)h\}] \cdots$$

$$\cdots [x-(a+h)] \quad \text{------ (2)}$$

Let $\dfrac{x-(a+mh)}{h} = m$

$\Rightarrow x = a+mh+mh$

$\therefore$ (2) $\Rightarrow$

$$f(a+mh+mh) = f(a+mh) + m\nabla f(a+mh) + \frac{m(m+1)}{\lfloor 2}\nabla^2 f(a+mh) + \cdots + \frac{m(m+1)\cdots(m+m-1)}{\lfloor m}\nabla^m f(a+mh)$$

This is Newton-Gregory formula for backward interpolation. This formula is used to interpolate the value of $f(x)$ when x is much greater than 'a' i.e. if the value to be interpolated lies on the other side.

Ex) The following table gives the sales of a concern for the last five years. Estimate the sales for the year 1951.

Years:	1946	1948	1950	1952	1954
Sales:	40	43	48	52	57

Soln:- The difference table is

Years	Sales	$\nabla f(x+mh)$	$\nabla^2 f(x+mh)$	$\nabla^3 f(x+mh)$	$\nabla^4 f(x+mh)$
1946	40				
		3			
1948	43		2		
		5		-3	
1950	48		-1		5
		4		2	
1952	52		1		
		5			
1954	57				

Here $x = 1951$, $a+mh = 1954$, $h=2$

$$\therefore m = \frac{x-(a+mh)}{h} = \frac{1951-1954}{2} = -\frac{3}{2} = -1.5$$

Now, by Newton's backward interpolation formula, we get,

$$f(a+mh+mh) = f(a+mh) + m\nabla f(a+mh) + \frac{m(m+1)}{\underline{|2}}\nabla^2 f(a+mh)$$

$$+ \frac{m(m+1)(m+2)}{\underline{|3}}\nabla^3 f(a+mh) + \frac{m(m+1)(m+2)(m+3)}{\underline{|4}}\nabla^4 f(a+mh)$$

$$\therefore f(1951) = 57 + (-1.5)\times 5 + \frac{(-1.5)(-1.5+1)}{\underline{|2}}\cdot 1$$

$$+ \frac{(-1.5)(-1.5+1)(-1.5+2)}{\underline{|3}}\times 2 + \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)}{\underline{|4}}\times 5$$

$$= 50.1172 \text{ (approx)}$$

Hence the sales for the year 1951 is 50.1172 //