Ex) A second degree polynomial passes through (0,1), (1,3), (2,7) and (3,13). Find the polynomial.

Solⁿ:- ∵ the polynomial $f(x)$ is of degree second,

$$\therefore \Delta^2 f(x) = \text{constant}$$

$$\text{so} \quad \Delta^3 f(x) = 0$$

Now, by Newton's forward interpolation formula, we have,

$$f(x) = f(a+mh) = f(a) + m\Delta f(a) + \frac{m(m-1)}{\underline{|2}} \Delta^2 f(a) + \frac{m(m-1)(m-2)}{\underline{|3}} \Delta^3 f(a) \quad \text{---- (1)}$$

The difference table is

x	$y=f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
		2		
1	3		2	
		4		0
2	7		2	
		6		
3	13			

Here, $a = 0$; $h = 1$

$$\therefore a + mh = x$$

$$\Rightarrow m = \frac{x-a}{h} = \frac{x-0}{1} = x$$

$$\therefore (1) \Rightarrow \quad f(x) = f(0) + x\,\Delta f(0) + \frac{x(x-1)}{\underline{|2}} \Delta^2 f(0) + \frac{x(x-1)(x-2)}{\underline{|3}} \Delta^3 f(0)$$

$$= 1 + x \cdot 2 + \frac{x(x-1)}{2} \cdot 2$$

$$\Rightarrow f(x) = x^2 + x + 1,$$ which is the reqd. polynomial.

Ex) A 3rd degree polynomial $f(x)$ passes through the pts (0,−1), (1,1), (2,1) and (3,−2).

(i) Find the polynomial : $-\frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{8}{3}x - 1$

(ii) Find the value at $x = 1{\cdot}2$: 1·192

Ex) Using Newton's forward interpolation formula, find the value of $f(22)$, from the following table:

x:	20	25	30	35	40	45
$f(x)$:	354	332	291	260	231	204

Ans:- $f(22) = 352.22$ (approx)

Ex) In an examination the number of candidates who obtained marks betn certain limits were as follows:

Marks:	No. of Candidates
0-19	41
20-39	62
40-59	65
60-79	50
80-99	17

Estimate the number of candidates who obtained fewer than 70 marks.

Ans: 196

Ex) Given

$\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$,

$\sin 60^\circ = 0.8660$.

Find $\sin 52^\circ$, by using any method of interpolation.

Soln: The table of finite difference is given below:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
45	0.7071			
		.0589		
50	0.7660		-.0057	
		.0532		-0.0007
55	0.8192		-.0064	
		.0468		
60	0.8660			

By Newton's interpolation formula, we have

$$f(a+mh) = f(a) + m\,\Delta f(a) + \frac{m(m-1)}{\underline{|2}}\Delta^2 f(a) + \frac{m(m-1)(m-2)}{\underline{|3}}\Delta^3 f(a)$$

Here, $a = 45^\circ$, $m = \frac{x-a}{h} = \frac{52-45}{5} = 1.4$

$\therefore \; f(52°) = 0.7071 + 1.4 \times .0589 + \frac{(1.4)(0.4)}{\lfloor 2} \times (-0.0057)$

$$+ \frac{(1.4)(0.4)(-0.6)}{\lfloor 3} \times (-0.0007)$$

$$= 0.780032$$

$\therefore \quad \sin 52° = 0.7880032$ //

※ Interpolation with unequal intervals:

Divided differences:

Consider the f^n $y = f(x)$. Let corresponding to the values $x_0, x_1, x_2, \ldots, x_{n-1}, x_n$ of the argument x, the values of the f^n $y = f(x)$ be $f(x_0), f(x_1), f(x_2), \ldots, f(x_{n-1}), f(x_n)$ where the intervals $x_1 - x_0, x_2 - x_1, \ldots x_n - x_{n-1}$ are not necessarily equal. Then $\frac{f(x_1) - f(x_0)}{x_1 - x_0}$ is called the first divided difference of $f(x)$ for the argument x_0, x_1 and is denoted by $f(x_0, x_1)$ or by $\Delta f(x_0)$ or by Δy_0.

Similarly, $f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

and so on.

The second order divided differences of $f(x)$ are defined in terms of the first order divided differences. Thus the second order divided differences of $f(x)$ for the arguments x_0, x_1, x_2 is defined by $\frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$ and denoted by $f(x_0, x_1, x_2)$ or by $\Delta^2 f(x_0)$ or by $\Delta^2 y_0$

Similarly, we have,

$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$ and so on.

The nth order divided difference for the arguments $x_0, x_1, x_2, \ldots x_n$ is defined as

$$f(x_0, x_1, x_2, \ldots, x_n) = \frac{f(x_1, x_2, \ldots, x_n) - f(x_0, x_1, \ldots x_{n-1})}{x_n - x_0}$$

Divided difference table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
x_0	$f(x_0)$			
		$f(x_0,x_1) = \dfrac{f(x_1)-f(x_0)}{x_1-x_0}$		
x_1	$f(x_1)$		$f(x_0,x_1,x_2) = \dfrac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$	
		$f(x_1,x_2) = \dfrac{f(x_2)-f(x_1)}{x_2-x_1}$		$f(x_0,x_1,x_2,x_3) = \dfrac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0}$
x_2	$f(x_2)$		$f(x_1,x_2,x_3) = \dfrac{f(x_2,x_3)-f(x_1,x_2)}{x_3-x_1}$	
		$f(x_2,x_3) = \dfrac{f(x_3)-f(x_2)}{x_3-x_2}$		$f(x_1,x_2,x_3,x_4) = \dfrac{f(x_2,x_3,x_4)-f(x_1,x_2,x_3)}{x_4-x_1}$
x_3	$f(x_3)$		$f(x_2,x_3,x_4) = \dfrac{f(x_3,x_4)-f(x_2,x_3)}{x_4-x_2}$	
		$f(x_3,x_4) = \dfrac{f(x_4)-f(x_3)}{x_4-x_3}$		
x_4	$f(x_4)$			
⋮	⋮			