

## **Unit 6.2 Rotational, Vibrational and Raman Spectroscopy (Marks 20)**

Rotational spectra of diatomic molecules – rigid rotator concept – determination of bond length – effect of isotopic substitution – spectra of non-rigid rotator. Vibrational spectra of diatomic molecules – harmonic and anharmonic oscillator model – Morse potential - calculation of force constants – effect of isotope - vibrations of polyatomic molecules, overtone and combination bands ( $\text{H}_2\text{O}$ ,  $\text{CO}_2$ ). Diatomic vibrating rotor – vibration rotation spectrum of CO. Principle of Raman spectroscopy – rotational and vibrational Raman spectra of linear molecules – rule of mutual exclusion.

Structure elucidation by IR spectroscopy – finger print region and group frequencies – effect of hydrogen bonding (alcohol, keto-enol) and coordination to metal.

# ROTATIONAL SPECTRA OF RIGID DIATOMIC MOLECULE (RIGID ROTATOR MODEL)

A rigid diatomic molecule means that the distance between the atoms (bond length) does not change during rotation. No vibrational movement is taking place during rotation.

Let us consider a diatomic molecule A—B in which the atoms A and B having masses  $m_1$  and  $m_2$  are joined together by a rigid bond of length  $r_0 = r_1 + r_2$  (figure 1). The molecule A—B rotates about a point C, the centre of gravity: this is defined by the moment, or balancing, equation. The moment of inertia about C is defined by

$$m_1 r_1 = m_2 r_2$$

...(1.1)

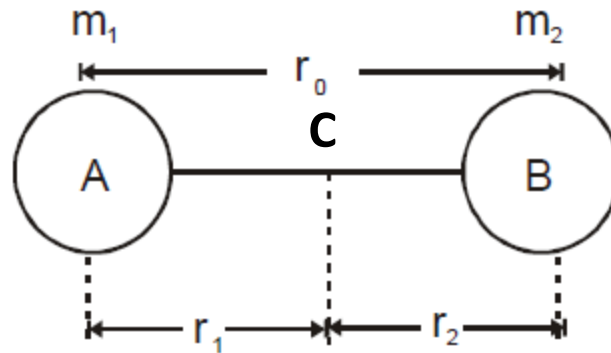


Fig. (1.1) : A rigid diatomic molecule A-B having atomic masses,  $m_1$  and  $m_2$ , joined together by a rigid bond of length  $r_0 = r_1 + r_2$  and rotates about a point C.

# ROTATIONAL SPECTRA OF RIGID DIATOMIC MOLECULE (RIGID ROTATOR MODEL)

$$I = m_1 r_1^2 + m_2 r_2^2 \quad \because [m_1 r_1 = m_2 r_2]$$

$$= m_2 r_2 r_1 + m_1 r_1 r_2$$

$$= r_2 r_1 (m_1 + m_2) \quad \dots(1.2)$$

from eq<sup>n</sup> (1)  $m_1 r_1 = m_2 r_2 = m_2 (r_0 - r_1)$

$$m_1 r_1 = m_2 r_0 - m_2 r_1$$

or  $m_1 r_1 + m_2 r_1 = m_2 r_0$

or  $r_1 (m_1 + m_2) = m_2 r_0$

$$\Rightarrow r_1 = \frac{m_2 r_0}{m_1 + m_2} \quad \text{Similarly} \quad r_2 = \frac{m_1 r_0}{m_1 + m_2} \quad \dots(1.3)$$

Putting the value of  $r_1$  &  $r_2$  from (3) in (2)

$$I = \frac{m_2 r_0}{(m_1 + m_2)} \frac{m_1 r_0}{(m_1 + m_2)} (m_1 + m_2) = \frac{m_1 m_2 r_0^2}{m_1 + m_2} = \mu r_0^2 \quad \dots(1.4)$$

where  $\mu$  is the reduced mass of the system i.e.

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad \dots(1.5)$$

# ROTATIONAL SPECTRA OF RIGID DIATOMIC MOLECULE (RIGID ROTATOR MODEL)

Equation (1.4) defines the moment of inertia conveniently in terms of the atomic masses and the bond length.

By the use of the Schrödinger equation it may be shown that the rotational energy levels allowed to the rigid diatomic molecule are given by the expression

$$E_J = \frac{h^2}{8\pi^2 I} J(J+1) \text{ Joules when } J = 0, 1, 2, \dots \quad \dots(1.6)$$

In this expression  $h$  is Planck's constant,  $I$  is the moment of inertia, either  $I_B$  or  $I_C$  since both are equal. The quantity  $J$ , which can take integral values from zero upwards, is called the rotational quantum number and each level is  $(2J + 1)$  fold degenerate.

# ROTATIONAL SPECTRA OF RIGID DIATOMIC MOLECULE (RIGID ROTATOR MODEL)

$$\varepsilon_J = \frac{E_J}{hc} = \frac{h}{8\pi^2 I c} J(J+1) \text{ cm}^{-1} \quad \left[ \frac{E}{hc} = \frac{1}{\lambda} \right]$$

$$= B J(J+1) \text{ cm}^{-1} \quad J = 0, 1, 2, \dots \quad \dots(1.7)$$

where B is the rotational constant and is given by

$$B = \frac{h}{8\pi^2 I c} \text{ cm}^{-1} = \frac{h^2}{8\pi^2 I} \text{ Joule}$$

Denoting the lower state by  $J''$  and the upper state by  $J'$

$$\Delta\varepsilon_J = \varepsilon_{J'} - \varepsilon_{J''}$$

$$= B J'(J'+1) - B J''(J''+1)$$

$$B [J'(J'+1) - J''(J''+1)]$$

use of the selection rule  $(\Delta J = \pm 1)$  or  $J' - J'' = 1$  gives the frequency of the absorption line as

$$v_J = \Delta\varepsilon_J = B [(J''+1)(J''+2) - J''(J''+1)]$$

$$B(J''+1)2 \quad \dots(1.8)$$

$$J \longrightarrow J+1$$

$$\varepsilon_{J+1} = B(J+1)(J+2)$$

$$\varepsilon_J = B J(J+1)$$

$$\overline{\Delta\varepsilon} = B(J+1)(J+2-J)$$

$$\Delta\varepsilon = 2B(J+1) \text{ cm}^{-1} \text{ where } J = 0, 1, 2, \dots$$

# ROTATIONAL SPECTRA OF RIGID DIATOMIC MOLECULE (RIGID ROTATOR MODEL)

The rotational constant  $B$  is assumed to be the same in both lower and upper rotational states and double prime is dropped from equn (8). The allowed energy levels of a rigid diatomic rotor are illustrated in Fig. 1.2(a). Thus a step-wise raising of the rotational transitions result in an absorption spectrum consisting of spectral lines with a separation of  $2B$ , that is at  $2B, 4B, 6B, \dots$  (Fig. 1.2(b)). The lowering of stepwise energy results in identical emission spectrum. Fig. (a) allowed energy levels of a rigid diatomic rotor showing electric dipole allowed transitions and (b) the resulting absorption spectrum.

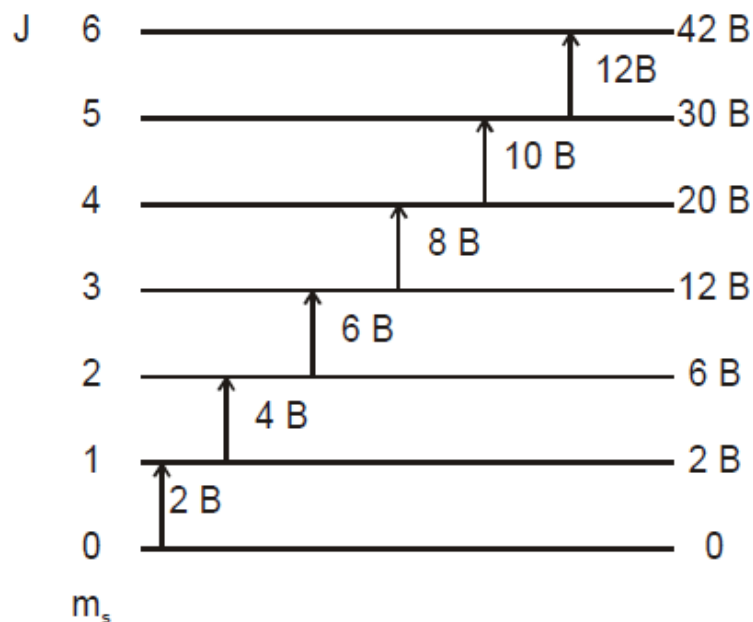


Fig. 1.2 (a)

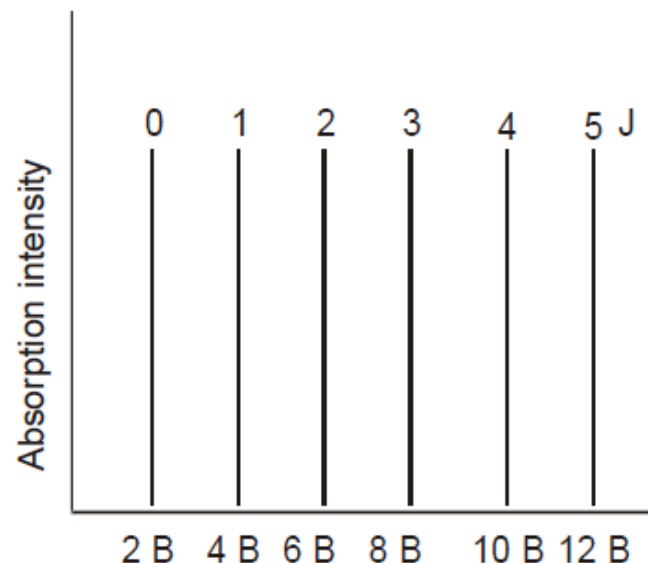


Fig. 1.2(b)

Fig. 1.2(a) allowed energy levels of a right diatomic rotor showing electric dipole allowed transitions and 1.2(b) the resulting absorption spectrum.

# Selection Rules

The selection rule for a transition between any two rotational states is quantum chemically given by

$$\mu_{ij} = \int \psi_i \mu \Psi_j d\tau$$

where  $\psi_i$  and  $\Psi_j$  are the wave functions for the rotational states  $i$  and  $j$  and  $\mu$  is the permanent dipole moment of the molecule. The dipole moment being a vector quantity can be expressed by its three components along the Cartesian coordinates axes,  $\mu = \vec{\mu}_x + \vec{\mu}_y + \vec{\mu}_z$  and  $\mu^2 = \mu_x^2 + \mu_y^2 + \mu_z^2$ . The transition moment integral can be split in terms of the components of the dipole moment,

$$\left[ \mu_{ij}^x \right] = \int \mu_i^x \mu_j d\tau$$

$$\left[ \mu_{ij}^y \right] = \int \mu_i^y \mu_j d\tau$$

$$\left[ \mu_{ij}^z \right] = \int \mu_i^z \mu_j d\tau$$

If atleast any one of the integrals is non zero, then the transition is allowed and forbidden otherwise. The intensity of an allowed rotational transition depends on the square of the

# Selection Rules

transition dipole moment. Consequently, the intensity of the rotational line depends on the square of the permanent dipole moment of the molecule.

Schrödinger equation shows that for a diatomic rigid rotor (in the absence of an external electric or magnetic field) only transitions in which  $J$  changes by one unit, that is,  $\Delta J = \pm 1$  are allowed and all other transactions are forbidden. Thus the selection rule for rotational spectra is  $\Delta J = \pm 1$  ( plus sign for absorption and minus sign for emission) and second, the molecule must have a permanent dipole moment (only heteronuclear diatomic molecules will exhibit the rotational spectrum since homonuclear diatomic molecules do not possess permanent dipole moment)



# NON RIGID ROTOR

From Schrödinger equation for a non-rigid rotator in simple harmonic force field.

$$E_J = \frac{h^2}{8\pi^2 I} J(J+1) - \frac{h^4}{32\pi^4 I^2 r^2 K} J^2 (J+1)^2 \text{ Joules} \quad K = \text{force constant}$$

$$\epsilon_J = \frac{E_J}{hc} = \frac{h^2}{8\pi^2 I hc} J(J+1) - \frac{h^4}{32\pi^4 I^2 r^2 K hc} J^2 (J+1)^2 \text{ cm}^{-1}$$

## Relationship between B & D

$$B = \frac{h^2}{8\pi^2 I hc} = \frac{h}{8\pi^2 I c}$$

$$K = 4\pi^2 c^2 \bar{\omega}^2 \mu$$

$$D = \frac{h^4}{32\pi^4 I^2 r^2 K hc} = \frac{h^3}{32\pi^4 I^2 r^2 K c} = \frac{h^3}{8 \times 2 \times 4 \times 8\pi^6 I^3 c^3} \frac{16\pi I c^2}{K r^2}$$

$$= B^3 \times \frac{16\pi^2 I c^2}{K r^2} = B^3 \times \frac{16\pi^2 \times \mu r^2 c^2}{4\pi^2 \bar{\omega}^2 c^2 \mu r^2} = \frac{B^3 \times 4}{\bar{\omega}^2} = \frac{4B^3}{\bar{\omega}^2}$$

$$\therefore \boxed{D = \frac{4B^3}{\bar{\omega}^2}}$$

$$[\because \frac{\omega}{\text{frequency}} = \frac{\bar{\omega}}{c}]$$

$\omega = \text{frequency}$

$\bar{\omega} = \text{wave number}$

$$\therefore \bar{\omega} = \frac{1}{2\pi c} \sqrt{\frac{k}{\mu}}$$

$$\therefore k = 4\pi^2 \bar{\omega}^2 m c^2]$$

$$[\because B^3 = \frac{h^3}{8 \times 8 \times 8\pi^6 I^3 c^3}]$$

$$[\because I = \mu r^2; k = 4\pi^2 \bar{\omega}^2 c^2 \mu]$$

# NON RIGID ROTOR

If  $B = 10 \text{ cm}^{-1}$  &  $\bar{\omega} = 10^3 \text{ cm}^{-1}$

$$D \approx \frac{(10 \text{ cm}^{-1})^3}{(10^3)^2} = \frac{10^3}{10^6} \approx 10^{-3} \text{ cm}^{-1}$$

So  $D \ll B$

*i.e.* D is much smaller than B. For small values of J, the correction term in equation (1.9) is negligibly small. For higher values of J, say 10 or more the value of D may be appreciable. The selection rule  $\Delta J = \pm 1$  is of course still valid. The spectral line given by the equation

$$B_J(J+1) - DJ^2(J+1)^2 \text{ cm}^{-1}$$

$$\epsilon_J = B_J(J+1) - DJ^2(J+1)^2 \text{ cm}^{-1}$$

# NON RIGID ROTOR

If  $D$  is neglected, the spectral lines occur at interval of  $2B$ .

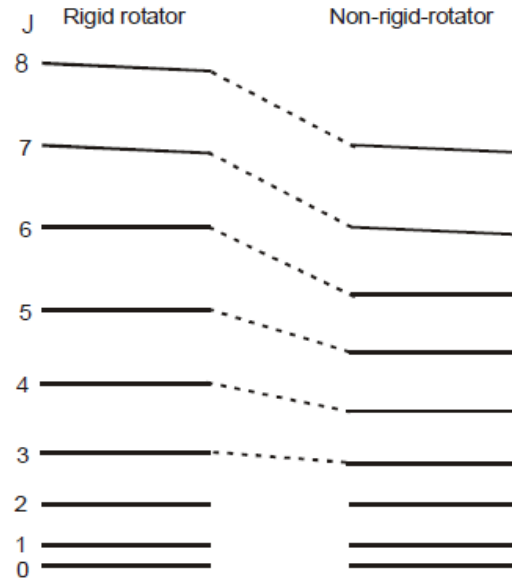


Fig. 1.3 (a)

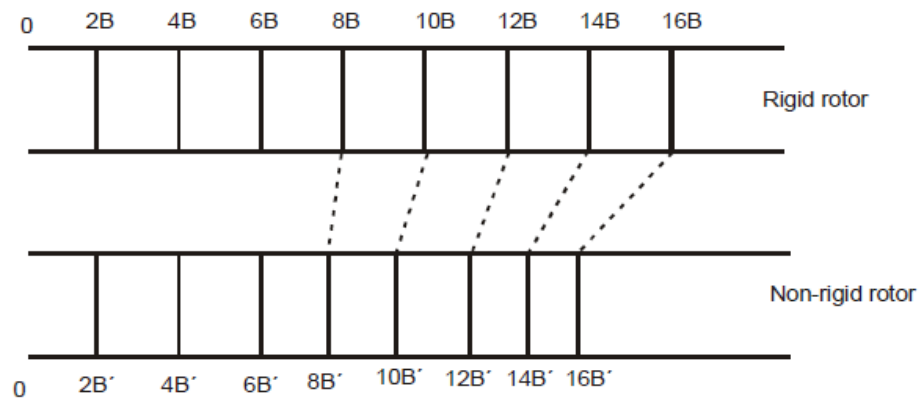


Fig. 1.3(b)

Fig. 1.3 (a) rotational energy levels and 1.3 (b) allowed spectral lines for rigid and non-rigid rotors.

# NON RIGID ROTOR

For the an harmonic oscillator the expression (1.9) should be modified as

$$\varepsilon_J = BJ(J+1) - DJ^2(J+1)^2 + HJ^3(J+1)^3 + KJ^4(J+1)^4 \text{ cm}^{-1} \quad \dots(1.10)$$

where the constants H, K etc, are dependent on the molecule. These constants H, K etc. are very small compared to D and hence can be neglected.

The value of D is also given by

$$D = \frac{16B^3\pi^2\mu c^2}{K} = \frac{4B^3}{\bar{\omega}^2}$$

# NON RIGID ROTOR

The spectral line  $\bar{\nu}_J$  is given by the equation

$$E_{J+1} - E_J = \bar{\nu}_J = 2B(J+1) - 4D(J+1)^3 \text{ cm}^{-1} \quad \dots(1.11)$$

If D is neglected, the spectral lines occur at intervals of 2B. The rotational spectrum of a diatomic molecule such as HF, CO etc. can be fitted to the equation (1.11).

Information from D

1. Determination of J value
2. Determination of vibrational frequency

$$D = \frac{4B^3}{\bar{\omega}^2}$$
$$\bar{\omega}^2 = \frac{4B^2}{D}$$

In HF  $B = 41.122 \text{ cm}^{-1}$

$$D = 8.52 \times 10^{-3} \text{ cm}^{-1}$$

$$\bar{\omega}^2 = \frac{4 \times (41.122)^3}{8.52 \times 10^{-3} \text{ cm}^{-1}} (\text{cm}^{-1})^3 = 16.33 \times 10^6 (\text{cm}^{-1})^2$$

$$\bar{\omega} = 4.050 \times 10^3 \text{ cm}^{-1} = 4050 \text{ cm}^{-1}$$

The accurate value is  $4138.3 \text{ cm}^{-1}$ . The 2% inaccuracy in the present calculation is due partly to the assumption of simple harmonic motion and partly to the very small and hence relatively inaccurate, value of D.

The force constant follow directly

$$K = 4\pi^2 c^2 \bar{\omega}^2 \mu = 960 \text{ Nm}^{-1}$$

which indicates, as expected, the H-F is a relatively strong bond.