* The shifting operation E :

The operator E defined by $E f(x) = f(x+h)$, where h is the interval of differencing, is called the shifting operator. By applying the operator E twice, we have

$$E^2 f(x) = E\{E f(x)\} = E f(x+h) = f(x+2h),$$

similarly,

$$E^3 f(x) = E\{E^2 f(x)\} = E f(x+2h) = f(x+3h)$$

and so on.

In general, $E^n f(x) = f(x+nh)$

Also we define E^{-1} by

$$E^{-1} f(x) = f(x-h) \text{ so that}$$

$$E^{-n} f(x) = f(x-nh)$$

* Properties of the operators E and Δ.

(i) operators E and Δ are distributive:

Consider a f^n $u(x)$ which is the sum of the f^ns $f(x)$, $g(x)$, ---- $P(x)$, -----.

i.e. $u(x) = f(x) + g(x) + P(x) + \cdots$

Then, $Eu(x) = f(x+h) + g(x+h) + P(x+h) + \cdots$

$\Rightarrow Eu(x) = Ef(x) + Eg(x) + EP(x) + \cdots$ [The interval of differencing is taken as h]

i.e. E is distributive

Again,

$$\Delta u(x) = \Delta \{f(x) + g(x) + P(x) + \cdots\}$$

$$= [f(x+h) + g(x+h) + P(x+h) + \cdots] - [f(x) + g(x) + P(x) + \cdots]$$

$$= \{f(x+h) - f(x)\} + \{g(x+h) - g(x)\} + \{P(x+h) - P(x)\} + \cdots$$

$$= \Delta f(x) + \Delta g(x) + \Delta P(x) + \cdots$$

which shows that Δ is also distributive.

(ii) E and Δ are commutative with regard to constants.

i.e., $E[c f(x)] = c E f(x)$

and $\Delta[c f(x)] = c \Delta f(x)$, when c is constant.

We have,

$$E[c f(x)] = c f(x+h)$$, the interval of differencing being h.

$$= c E f(x)$$

Again, $\Delta[c f(x)] = c f(x+h) - c f(x)$

$$= c \{f(x+h) - f(x)\}$$

$$= c \Delta f(x)$$

(iii) E and Δ obey the law of indices.

i.e., $E^m E^n f(x) = E^{m+n} f(x)$

and $\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$

We have,

$$E^m E^n f(x)$$

$$= E^m f(x+nh)$$

$$= f(x+mh+nh)$$

$$= f[x+(m+n)h]$$

$$= E^{(m+n)} f(x).$$

Similarly, we can show that

$$\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$$

Note:

The operators Δ and E are not commutative with respect to the functions of x i.e. (variables)

if $u_x = f(x) \cdot g(x)$, then

$$\Delta u_x \neq f(x) \cdot \Delta g(x)$$

and $$E u_x \neq f(x) \cdot E g(x)$$

(iv) $E\Delta \equiv \Delta E$

We have,
$$\Delta E f(x) = \Delta[E f(x)]$$
$$= \Delta[f(x+h)]$$
$$= f(x+2h) - f(x+h)$$

and
$$E\Delta f(x) = E[\Delta f(x)]$$
$$= E[f(x+h) - f(x)]$$
$$= E f(x+h) - E f(x)$$
$$= f(x+2h) - f(x+h)$$

$\therefore$ $\Delta E \equiv E\Delta$

(v) $$\Delta[f(x) g(x)] = f(x)\Delta g(x) + g(x+h)\Delta f(x)$$

L.H.S. $= \Delta[f(x) g(x)]$
$$= f(x+h) g(x+h) - f(x) g(x)$$
$$= g(x+h) f(x) - f(x) g(x) + f(x+h) g(x+h) - g(x+h) f(x)$$
$$= f(x)[g(x+h) - g(x)] + g(x+h)[f(x+h) - f(x)]$$
$$= f(x)\Delta g(x) + g(x+h)\Delta f(x)$$
$= R.H.S.$

//

(vi)
$$\Delta\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\,\Delta f(x) - f(x)\,\Delta g(x)}{g(x)\,g(x+h)}$$

L.H.S.

$$\Delta\left[\frac{f(x)}{g(x)}\right]$$

$$= \frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}$$

$$= \frac{f(x+h)\,g(x) - f(x)\,g(x+h)}{g(x)\,g(x+h)}$$

$$= \frac{[g(x)\,f(x+h) - f(x)\,g(x)] - [f(x)\,g(x+h) - f(x)\,g(x)]}{g(x)\,g(x+h)}$$

$$= \frac{g(x)\,[f(x+h) - f(x)] - f(x)\,[g(x+h) - g(x)]}{g(x)\,g(x+h)}$$

$$= \frac{g(x)\,\Delta f(x) - f(x)\,\Delta g(x)}{g(x)\,g(x+h)} = \text{R.H.S.}$$

(vii) $E^0 f(x) = f(x)$

(viii) $E^{-m} f(x) = f(x-mh)$

(ix) $\Delta K = 0$

(x) $EK = K$ where K is a constant

(xi) $E^2 f(x) \neq \{E f(x)\}^2$